Non-Euclidean Electromagnetic Kerr Model for Hydrogen

Abstract

A Balmer series of observed hydrogen data was compared to two geometric levels of atomic theory and modeling. The first theory compared was the Euclidean-based, special relativistic Dirac theory, with QED corrections added. The second theory applied was a non-Euclidean electromagnetic Kerr field theory with Euclidean QED corrections added. Each model was used to predict the Balmer series transition wavelengths, and then compared to the observed data. The statistics for the model performances were computed, and show a noticeable increase in accuracy and precision of the model predictions using the non-Euclidean Kerr field theory with QED compared to Euclidean Dirac theory with QED. These results suggest Euclidean Dirac theory is too restrictive/special and does not incorporate an important relativistic contributor. According to non-Euclidean field theory, the time dilation the electron experiences is a function of not only its velocity (as in Dirac theory), but time dilation is also a function of its position in the "generalized" electromagnetic Kerr field of the hydrogen atom. The time dilation the electron experiences, is then stronger than in Dirac special relativity. The introduction of these added non-Euclidean relativistic effects produces predictions in better agreement with the observed hydrogen Balmer data.

The Hydrogen Balmer Data

A set of observed Balmer series hydrogen data was obtained from the text book: *The Physics of Atom and Quanta,* H. Haken, H. Wolf and W. Brewer, 6th Edition, 2004, Springer-Verlag. The data set from the text is:

Table 8.2. The first 20 lines of the Balmer series of hydrogen. The numbers quoted are wavelengths in air, the wavenumbers in vacuum, and the values calculated from the Balmer formula

n	λ_{air} [Å]	$\tilde{\nu}_{vac}$ [cm^{-1}]	$R_H\left(\frac{1}{2^2}-\frac{1}{n^2}\right)$
H_α 3	6562.79	15233.21	15233.00
H_β 4	4861.33	20564.77	20564.55
H_γ 5	4340.46	23032.54	23032.29
H_δ 6	4101.73	24373.07	24372.80
H_ε 7	3970.07	25181.33	25181.08
H_ζ 8	3889.06	25705.84	25705.68
H_η 9	3835.40	26065.53	26065.35
H_ϑ 10	3797.91	26322.80	26322.62
H_ι 11	3770.63	26513.21	26512.97
H_κ 12	3750.15	26658.01	26657.75
H_λ 13	3734.37	26770.65	26770.42
H_μ 14	3721.95	26860.01	26859.82
H_ν 15	3711.98	26932.14	26931.94
H_ξ 16	3703.86	26991.18	26990.97
H_o 17	3697.15	27040.17	27039.89
H_π 18	3691.55	27081.18	27080.88
H_ϱ 19	3686.83	27115.85	27115.58
H_σ 20	3682.82	27145.37	27145.20

The (in vacuo) Balmer series data in this text are in wavenumbers (in cm^{-1}), which were converted to wavelengths in meters, for transitions from main shells $n = 3$ to $n = 2$, then $n = 4$ to $n = 2$, etc., up to $n = 20$ to $n = 2$. (18 data points.) The observed Balmer data are astrophysical, taken using astronomical spectroscopy. Assuming theory is correct, there must have been numerous (n) subshell -to- ($n = 2$) subshell transitions involved in these data. But only a single transition data point is listed for each Balmer transition. A single transition data value was assumed to be obtained from a weighted average, with weights based on the observed subshell-to-subshell transition intensities. Past historical hydrogen lab spectroscopy shows the "humps" seen around a single transition were averaged with intensity weights to provide a single transition value. Or if the spectroscopic machine had relatively poor resolution (but apparently it was pretty good, as will be shown), nature itself weighted the single observed line.

Euclidean Dirac Theory Plus QED Results

Ignoring hyperfine splitting, there are 3 sublevels for $n = 2$, the outer $2P_{3/2}$ ($l = 1$, magnetic) sublevel with QED, then the mid spherical $2S_{1/2}$ ($l = 0$, nonmagnetic) sublevel with QED, then the lowest $2P_{1/2}$ ($l = 1$, magnetic) sublevel with QED. In basic Dirac theory, the last two (without QED) are degenerate. Introducing QED effects/corrections "breaks the degeneracy." The paper

at the following link provides the needed theoretical hydrogen subshell energies to use for a Balmer series predicition, with QED corrections:

http://www.nist.gov/data/PDFfiles/jpcrd100.pdf

The above NIST paper lists (p. 853) all of the predicted hydrogen subshell energies (in cm^{-1}) with QED corrections for n = 1, 2, …, 8 and further. It is an older paper (1977), but the paper's predicted values agree well with the current values listed at NIST. To get the $2P_{3/2}$, $2S_{1/2}$ and $2P_{1/2}$ Dirac + QED predicted sublevels for the Balmer transitions, the values listed in the paper were used. For the analysis here, the "plus QED" subshell energies also listed in the paper for the Balmer series predictions for n = 3, …, 8 were used. The basic Dirac equation (without QED) was then used to compute predictions for n = 9, 10, … , 20. The basic Dirac equation used was eq. (2.4) of the above paper, listed here:

$$E_{n,j} = \mu c^2 \left[1 + \left(\frac{\alpha}{n - \varepsilon} \right)^2 \right]^{-1/2} - \mu c^2$$

where $\varepsilon = j + 1/2 - [(j + 1/2)^2 - \alpha^2]^{1/2}$ and μ is the electron's reduced rest mass in hydrogen. For the outer main shells, the QED corrections get very small, and no significant error is hence introduced if the QED corrections are omitted for these outer shells (9 to 20).

Given the transition rules $\Delta j = 0, \pm 1, \Delta l = \pm 1$, the allowed transitions to the three n = 2 sublevels are

$nP_{1/2}$ <-> $2S_{1/2}$
$nP_{3/2}$ <-> $2S_{1/2}$
$nS_{1/2}$ <-> $2P_{1/2}$
$nD_{3/2}$ <-> $2P_{1/2}$
$nD_{5/2}$ <-> $2P_{1/2}$
$nS_{1/2}$ <-> $2P_{3/2}$
$nD_{3/2}$ <-> $2P_{3/2}$
$nD_{5/2}$ <-> $2P_{3/2}$

The procedure just described was used to predict the (18) transition wavelengths for each of the above transitions, using the NIST paper's subshell QED-corrected energy values for n = 2, …, 8, and basic Dirac predictions without QED for n = 9, …, 20. The predictions were then compared to the observed data. For the $nP_{1/2}$ <-> $2S_{1/2}$ transition, the results were

$n\underline{P_{1/2}} <-> \underline{2S_{1/2}}$
Sample average prediction error = +2.67999673 x 10^{-12} m
Sample standard deviation = 6.24440238 x 10^{-13} m
Standard error (the above divided by $\sqrt{18}$) = 1.47181976 x 10^{-13} m
5-sigma confidence interval = (+1.94408685 x 10^{-12},+3.41590661 x 10^{-12}) m

For this analysis, the individual observed, predicted, and prediction errors (wavelengths in m) were :

n = 3	6.56460457E-007	6.56458428E-007	2.02947188E-012
n = 4	4.86268507E-007	4.86265562E-007	2.94506857E-012
n = 5	4.34168355E-007	4.34165816E-007	2.53871789E-012
n = 6	4.10288897E-007	4.10286800E-007	2.09692431E-012
n = 7	3.97119612E-007	3.97117175E-007	2.43736638E-012
n = 8	3.89016659E-007	3.89012797E-007	3.86230807E-012
n = 9	3.83648443E-007	3.83644966E-007	3.47726963E-012
n = 10	3.79898795E-007	3.79895380E-007	3.41463844E-012
n = 11	3.77170475E-007	3.77167950E-007	2.52416634E-012
n = 12	3.75121774E-007	3.75119596E-007	2.17801972E-012
n = 13	3.73543414E-007	3.73540825E-007	2.58900413E-012
n = 14	3.72300680E-007	3.72297544E-007	3.13686476E-012
n = 15	3.71303580E-007	3.71300543E-007	3.03669394E-012
n = 16	3.70491398E-007	3.70488537E-007	2.86076901E-012
n = 17	3.69820160E-007	3.69818252E-007	1.90774570E-012
n = 18	3.69260128E-007	3.69258413E-007	1.71552799E-012
n = 19	3.68787997E-007	3.68785943E-007	2.05362119E-012
n = 20	3.68386948E-007	3.68383512E-007	3.43576319E-012

The observed data (converted from the wavenumbers in the referenced text book) are in the first data column, the predictions are in the second column, and the differences (errors) are in the third (last) column. The magnitudes of the prediction errors (observed wavelength minus predicted wavelength) are ~10^{-12} m. The transition wavelengths themselves are ~10^{-7} m for the Balmer series, so the prediction errors are about 5 orders of magnitude smaller than the observed variable magnitude. This indicates both the model and the data are reasonably accurate and precise. The precision in the fit even allows for an evaluation of a/this model's bias (inaccuracy).

For the above, all 18 prediction errors were positive, indicating a model bias. The positive prediction errors say this analysis produced predicted transition wavelengths that are too short as compared to "the truth" given by the data. Some type of bias is to be expected, since the observed data involved more transitions than just this one. Using only this single transition, this "not complete" model represents a "too inflated" hydrogen atom relative to the "correct compression" manifest in the observed data. The predicted wavelengths were too small (observed > predicted) and too energetic, saying the inter subshell-to-subshell delta transition

energies were too large and "inflated" compared to the data. The 5-sigma confidence interval for the true mean prediction error also does not contain zero, indicating a model bias. As mentioned, this is to be expected, since the observed data undoubtedly contain other transitions involving stronger spin-orbit magnetic effects than does this single transition.

The analysis was repeated for the other allowed Balmer transitions, producing the following results (values in m):

Transition	Sample Average Prediction Error	Sample Standard Deviation
$nP_{1/2}$ <-> $2S_{1/2}$	$+2.67999673 \times 10^{-12}$	$6.24440238 \times 10^{-13}$
$nP_{3/2}$ <-> $2S_{1/2}$	$+3.06225625 \times 10^{-12}$	$1.12816117 \times 10^{-12}$
$nS_{1/2}$ <-> $2P_{1/2}$	$+3.30505964 \times 10^{-12}$	$6.43759806 \times 10^{-13}$
$nD_{3/2}$ <-> $2P_{1/2}$	$+3.65092165 \times 10^{-12}$	$1.34348622 \times 10^{-12}$
$nD_{5/2}$ <-> $2P_{1/2}$	$+3.77833809 \times 10^{-12}$	$1.67893344 \times 10^{-12}$
$nS_{1/2}$ <-> $2P_{3/2}$	$-2.80498295 \times 10^{-12}$	$2.44089475 \times 10^{-12}$
$nD_{3/2}$ <-> $2P_{3/2}$	$-2.45910605 \times 10^{-12}$	$1.50638804 \times 10^{-12}$
$nD_{5/2}$ <-> $2P_{3/2}$	$-2.33168411 \times 10^{-12}$	$1.18493865 \times 10^{-12}$

For each case with a positive average, all prediction errors were also positive. For those with a negative average, all prediction errors were negative. A negative result describes a model that is "too compressed" relative to the data. The predicted wavelengths are too large (observed < predicted), relating back to a too small subshell-to-subshell delta-energy difference. This type of bias is "too compressed" compared to the data, and is also to be expected for particular single transition models.

An observed data value comes from a blend of all of these transitions, with the value of the single shell-to-shell value equal to a transition-intensity weighted average. The predicted weights could be computed to produce a single shell predicted value, and a better single shell prediction obtained to compare to the data. But, for comparison to the next atomic model, seeing the Euclidean Dirac + QED results for these separated-out transitions will be informative.

Non-Euclidean Eelectromagnetic Kerr Theory Results

The next atomic model used to compare to the hydrogen data is based on non-Euclidean spacetime differential geometries. For spherical subshells (n, $l = 0$, $j = ½$), the time dilation the electron experiences in this (special case) spherical non-Euclidean field theory is given as

$$\frac{dt}{d\tau}=\left(1-\left(\frac{v_n}{c}\right)^2-\frac{r_S}{r_n}\right)^{-1/2} \tag{1}$$

where the *electronic Schwarschild radius* (defined by the extended Gravitoelectromagnetic Equivalence Principle) is $r_S=\left|e_e e_p\right|/2\pi\varepsilon_0 m_e c^2=2\alpha\hbar/m_e c$ where m_e is the electron's rest mass, not its reduced rest mass. But as in Euclidean Dirac theory, the reduced rest mass μ of the electron must be used in the energy equations. (Please see the Appendix for a derivation of the electronic Schwarzschild radius r_S.) This time dilation (1) is electronic only in form, per the definition of r_S, which shows it is only defined for a system of two charges (and conforms to muonic hydrogen, e.g.). Note the mass and charge of the electron themselves (and the charge of the proton) set the basic non-Euclidean Schwarzschild structure of the field within which the electron orbits. Hence, if the mass of the orbiting body changes, for example, like in muonic hydrogen, the entire metric structure shifts to accommodate the new two-body system. A main effect of using a non-Euclidean field theory, is the introduction of the position-dependent field contribution to the total time dilation, with the necessary inclusion of the r_S/r_n term. The time dilation is hence greater than in special relativity.

The Schwarzschild electron orbital radius and velocity (r_n and v_n) in the n^{th} spherical subshell (needed in the time dilation (1)) are initially given by

$$\begin{aligned} r_n &= r_B\frac{d\tau}{dt} \\ v_n &= v_B\left(\frac{dt}{d\tau}\right)^{1/2} \end{aligned} \tag{2}$$

where r_B and v_B are the nonrelativistic Bohr radius and velocity for the respective n^{th} spherical main shell. Using Bohr values provides only initial approximate values for the needed Schwarzschild r_n and v_n and the non-Euclidean time dilation. Due to the nonlinear nature of non-Euclidean field theory, numerical iterations are useful, and sometimes required. Double precision numerical iteration to convergence provides essentially theoretically exact results. To compute a correct value of the Schwarzschild time dilation for a given n, the values of r_n and v_n are initialized at their Bohr values (eq. 2) (initialize the time dilation to 1), and then the time dilation (eq. 1) is updated. Next, the updated time dilation is used to compute new Schwarzschild relativistic values of r_n and v_n, per (2). These updated radius and velocity values are then reinserted into the time dilation eq. (1), and the iteration continued until convergence. Note therefore, time dilation is quantized, as it is in Euclidean Dirac theory.

Given the converged values of orbital radius, velocity and time dilation, the non-Euclidean electronic spherical (Schwarzschild) total orbital energy the electron experiences is given as

$$E_n = \frac{\mu c^2}{2}\left(\left[\left(1-\frac{r_S}{r_n}\right)\frac{dt}{d\tau}\right]^2 - 1\right) \tag{3}$$

where now the electron's reduced rest mass μ resides in this equation. While equation (3) looks completely gravitational in identity (allowed by the extended Gravitoelectromagnetic Equivalence Principle) it is in fact completely electronic in form, and specific to a system of bound charges (here, hydrogen). Nowhere in this equation does Newton's gravitational constant *G* appear.

These equations are specific for the exact spherical symmetry of a Schwarzschild geometry, so while not explicitly in the equations, the Dirac quantum numbers are j = ½ and l = 0 (true S subshell). The Dirac quantum number s is also unimportant here. No electron-spin-magnetic effects are present due to the complete spherical symmetry of a Schwarzschild field.

To incorporate non-Euclidean spin-orbit magnetic effects, the differential geometry can be generalized to that of Kerr. (In the gravitational world, this predicts gravitomagetism, as was successfully observed by Gravity Probe B in orbit about the spinning mass of the Earth.) The extended Gravitoelectromagnetic (GEM) Equivalence Principle allows electromagnetism to be represented as a type of non-Euclidean electromagnetic Kerr "frame dragging," which suggests a "unification" of "electro-" and "gravito-" magnetisms.

The electromagnetic Kerr 4 x 4 timelike metric tensor, in spherical polar spacetime coordinates $\mathbf{x} = (r,\theta,\phi,t)^T$, is (dropping orbital indexing for now):

$$\mathbf{G} = \begin{bmatrix} g_{rr} & 0 & 0 & 0 \\ 0 & g_{\theta\theta} & 0 & 0 \\ 0 & 0 & g_{\phi\phi} & g_{\phi t} \\ 0 & 0 & g_{t\phi} & g_{tt} \end{bmatrix} \tag{4}$$

where

$$g_{rr} = -\frac{\Sigma}{c^2 \Delta}, \quad g_{\theta\theta} = -\frac{\Sigma}{c^2}, \quad g_{\phi\phi} = -\frac{1}{c^2}\left[\frac{r^2 + a^{2\ 2} - \Delta a^2 \sin^2\theta}{\Sigma}\right]\sin^2\theta,$$

$$g_{tt} = \frac{\Delta - a^2 \sin^2\theta}{\Sigma}, \quad \text{and} \quad g_{t\phi} = g_{\phi t} = \frac{1}{c^2}\left[\frac{a \sin^2\theta \ \ r^2 + a^2 - \Delta}{\Sigma}\right]$$

with

$$\Sigma = r^2 + a^2 \cos^2\theta, \quad \text{and} \quad \Delta = r^2 + a^2 - r_S r$$

Along with the electronic Schwarzschild r_S, the electromagnetic Kerr "frame dragging parameter *a*" enters the equations The final forms of these equations are completely electromagnetic. Newton's *G* does not appear anywhere. It should also be stressed, especially so here, the "magnetisim" modeled here is "central" in that it is accorded to the "spin of a central body," as easily modeled with non-Euclidean field theory. This is the analog in Euclidean theory, where the electron's current-loop orbital motion sets up an "effective centrally located" magnetic N-S dipole, creating the "central" magnetic field within which the electron orbits. This is also the basis for the spin-orbit interaction. Here, in the non-Euclidean field theory, the centrality of the dominant magnetism is also maintained, with the frame dragging parameter *a* defined as proportional to the "spin" of a "central body."

For any generalized 4-D spacetime geometry, the generalized time dilation equals

$$\frac{dt}{d\tau} = \left(\frac{d\mathbf{x}^T}{dt}\mathbf{G}\frac{d\mathbf{x}}{dt}\right)^{-1/2} \tag{5}$$

(Equation (5) is obtained by rewriting the usual form of the metric, with $d\tau^2$ on the left.) Expansion of the quadratic form for a Kerr geometry shows, for equatorial circular obits, the Kerr electromagnetic time dilation simplifies to

$$\frac{dt}{d\tau} = \left(1 - \left(\frac{v_n}{c}\right)^2\left(1 + \frac{a_n^2}{r_n^2} + \frac{r_S a_n^2}{r_n^3}\right) - \frac{r_s}{r_n} + 2\frac{v_n}{c^2}\frac{a_n r_S}{r_n^2}\right)^{-1/2} \tag{6}$$

where subindexing by the main shell number *n* has been reintroduced. The correct value of a_n is to be determined shortly.

At this point, the rest of the full set of Dirac quantum numbers *n*, *j*, *l* and *s* are inserted into in the equations as follows. The principle magnetic interaction in hydrogen is the spin-orbit magnetic

interaction, the value of which is "directed" by full set of Dirac quantum numbers. Deterministic quantum theory shows the spin-orbit delta-energy of orientation, of the (anomalous) magnetic moment of the electron is

$$\begin{aligned} \Delta E_{so} &= g_e \frac{\hbar^2}{4\mu^2 c^2}[j(j+1) - l(l+1) - s(s+1)]\frac{1}{r_n}\frac{dV(r_n)}{dr_n} \\ &= g_e \frac{\hbar^2}{4\mu^2 c^2}[j(j+1) - l(l+1) - s(s+1)]\frac{\left|e_e e_p\right|^2}{4\pi\varepsilon_0 r_n^3} \end{aligned} \tag{7}$$

The full set of Dirac quantum numbers reside in (7). With the rules $\Delta E_{so} = 0$ for j = ½ regardless of l, and s always equals ½, the Dirac quantum numbers direct the sign and magnitude of the spin-orbit delta-energy. The observed anomalous electron magnetic "g-factor" $g_e = 2.0023193$ was incorporated into the computations/predictions. Note that the addition of the Thomas precession of Euclidean special relativity has not been included. (If it were, (7) would be multiplied by ½.) In non-Euclidean field theory, such a precession is not warranted. The authors MTW in their text book "Gravitation" state on (the bottom of) p. 1118: "The Thomas precession comes into play for a gyroscope on the surface of the Earth (***a*** = Newtonian acceleration of gravity), but not for a gyroscope in a freely moving satellite." If non-Euclidean modeling is used, as done here, then no Thomas precession should be incorporated. The mathematical physics is explained by MTW. The scaling of the problem is irrelevant. For both macroscopic or microscopic geodesic motion, if differential geometry is used to model the coasting motion of gyroscopes (such as the electron), no Thomas precession should be included in the theory. Also, strictly speaking, the use of only a classic Coulomb potential is not general enough in this non-Euclidean field theory, but the above energy will be involved in the time dilation numerical convergence, which reduces the error in theory. From a probabilistic "expectation" standpoint, the use of the deterministic inverse of r_n^3 is tantamount to using a "maximum pdf/maximum mode" population moment, also a valid measure of "central tendency" in mathematical statistical theory.

To incorporate electromagnetic Kerr frame dragging effects, the electromagnetic Kerr total orbital energy shows the route. This energy is generally (for all orbits, not just circular equatorial)

$$E_n = \frac{\mu c^2}{2}\left[\left(\left(1 - \frac{r_S r_n}{\Sigma}\right)\frac{dt}{d\tau} + \frac{r_S a_n r_n \sin^2\theta}{\Sigma}\frac{d\phi}{d\tau}\right)^2 - 1\right] \tag{8}$$

An important character of the structure of the Kerr energy equation is its "perturbation" nature, right in the equation. The aspherical magnetic term involving *a* is simply added to the spherical Schwarzschild term.

For equatorial circular orbits, the Kerr energy equation simplifies to

$$E_n = \frac{\mu c^2}{2}\left[\left(\left(1-\frac{r_S}{r_n}\right)\frac{dt}{d\tau} + r_S a_n \frac{v_n}{r_n^2}\frac{dt}{d\tau}\right)^2 - 1\right] \tag{9}$$

An expansion of (9) separates the energy into individual nonmagnetic and magnetic terms. An isolation of the terms involving the magnetic spin-orbit a_n produces

$$\Delta E_{so} = \mu c^2\left(1-\frac{r_S}{r_n}\right)\left(\frac{dt}{d\tau}\right)^2 \frac{r_S v_n}{r_n^2} a_n + 2\mu c^2\left(\frac{r_S v_n}{2r_n^2}\right)^2\left(\frac{dt}{d\tau}\right)^2 a_n^2 \tag{10}$$

Please note, while not subindexed as so, the quantized time dilation is also a function of *n*. Also, the r_n and v_n are now even more "general" than in the Schwarzschild theory; they are now the quantized Kerr orbital radii and velocities. Computational results for hydrogen show, though, the converged-upon Kerr radii and velocities are very nearly equal to their Schwarschild values.

The extended GEM Equivalence Principle allows setting equality between equations (7) and (10). Setting these equations equal results in

$$g_e \frac{\hbar^2}{4\mu^2 c^2}[j(j+1) - l(l+1) - s(s+1)]\frac{\left|e_e e_p\right|^2}{4\pi\varepsilon_0 r_n^3} = \mu c^2\left(1-\frac{r_S}{r_n}\right)\left(\frac{dt}{d\tau}\right)^2 \frac{r_S v_n}{r_n^2} a_n + 2\mu c^2\left(\frac{r_S v_n}{2r_n^2}\right)^2\left(\frac{dt}{d\tau}\right)^2 a_n^2 \tag{11}$$

Equation (11) is a parabola in a_n, and has all Dirac quantum numbers correctly incorporated. The solution for a_n is found by root taking. Set

$$\beta_2 = 2\mu c^2 \left(\frac{r_S v_n}{2r_n^2}\right)^2 \left(\frac{dt}{d\tau}\right)^2$$

$$\beta_1 = \mu c^2 \left(1 - \frac{r_S}{r_n}\right)\left(\frac{dt}{d\tau}\right)^2 \frac{r_S v_n}{r_n^2}$$

$$\beta_0 = -g_e \frac{\hbar^2}{4\mu^2 c^2}[j(j+1) - l(l+1) - s(s+1)]\frac{\left|e_e e_p\right|^2}{4\pi\varepsilon_0 r_n^3}$$

The parabola is now

$$\beta_2 a_n^2 + \beta_1 a_n + \beta_0 = 0$$

so that

$$a_{n,j,l,s} = \frac{-\beta_1 + \sqrt{\beta_1^2 - 4\beta_2\beta_0}}{2\beta_2} \tag{12}$$

Numerical computations show the positive root has the physically correct sign. The full set of Dirac quantum numbers now reside in the quantized electromagnetic Kerr total orbital energies. When l = 0, j must equal ½, and $a_{n,j,l,s}$ equals zero (magnetic $\beta_0 = 0$). The field theory then drops to the nonmagnetic Schwarzschild (spherical) field theory as the special case. The two electromagnetic "characteristic lengths" r_S and $a_{n,j,l,s}$ (devoid of G) enter the Kerr metric, and completely specify the structure of the Kerr electromagnetic spacetime for given values of the Dirac quantum numbers. Note this model is "adaptive" in the sense each sublevel has its own electromagnetic Kerr metric structure, as $a_{n,j,l,s}$ (the frame dragging, i.e., the spin-orbit magnetism) discretely changes from subshell to subshell. As in the Schwarzschild case, the added nonlinear field effects change the time dilation. The frame dragging parameter $a_{n,j,l,s}$ must enter the time dilation iteration, and then both converge on their correct Kerr values, now with all Dirac quantum numbers directing the convergence.

Euclidean QED effects can be inserted into the non-Euclidean theory by adding to β_0. The dominant electron self energy and vacuum polarization effects, plus additional spin-orbit effects where appropriate (l not 0, j not 1/2), were included by computing QED delta-energies using eqs. (2.10) and (2.11) of the NIST paper. These were combined as

$$\Delta E_{QED} = \frac{4\alpha(Z\alpha)^4 m_e c^2}{3\pi n^3}\left(\frac{\mu}{m_e}\right)^3\left[\left(\log 10\frac{1}{(Z\alpha)^2} + \log 10\frac{m_e}{\mu} + \frac{11}{24} + \frac{3}{8} - \frac{1}{5} + L_n\right)\delta_{l0} + \left(\frac{3}{8}\frac{C_{lj}}{2l+1} + L_n\right)(1-\delta_{l0})\right] \qquad (13)$$

For hydrogen, $Z = 1$. The Dirac delta function is: $\delta_{l0} = 1$ if $l = 0$ and $\delta_{l0} = 0$ if l not zero. The correct Bethe logarithm value of L_n was obtained from Appendix B of the NIST paper (p. 846). The coefficients C_{lj} are obtained from

$$C_{lj} = \begin{pmatrix} 1/(l+1) & \text{for } j = l + 1/2 \\ -1/l & \text{for } j = l - 1/2 \end{pmatrix}$$

To incorporate these Euclidean QED effects, eq. (13)'s value was added to the frame dragging β_0 as

$$\beta_0 = -(\Delta E_{so} + \Delta E_{QED})$$

where ΔE_{so} is given by (10). This forces convergence to an aspherical Kerr structure, even for the "truly spherical" QED-effected $n\mathrm{S}_{1/2}$ subshells in Euclidean Dirac theory. In this non-Euclidean theory, the only way to model off-spherical (off-Schwarzschild) subshells is to introduce frame dragging, using the more general magnetic Kerr theory. In this atomic EM Kerr theory, there are no "truly spherical" subshells, when QED effects necessarily must enter as a type of "magnetic frame dragging" in an atomic Kerr field.

The EM Kerr theory predicts the correct observed Lamb shift for the $2\mathrm{S}_{1/2}$ <-> $2\mathrm{P}_{1/2}$ transition, a value of about 1057.9 MHz. It certainly should, since the known QED equations (which work) were used to "perturb" the Kerr theory through the use of a frame dragging "magnetic Kerr QED perturbation." It may be that the Euclidean QED effects/corrections can (and should) be generalized to this non-Euclidean theory, but certainly to at least "first order," utilizing Euclidean QED corrections should suffice.

The electromagnetic Kerr field theory plus Euclidean QED effects, was used to compute subshell energies and energy differences, and non-Euclidean Kerr predicted transition wavelengths. The results for the various non-Euclidean Kerr transitions were:

Transition	Sample Average Prediction Error	Sample Standard Deviation
$nP_{1/2}$ <-> $2S_{1/2}$	-1.41541844 x 10^{-12}	9.90931632 x 10^{-13}
$nP_{3/2}$ <-> $2S_{1/2}$	-1.39289224 x 10^{-12}	9.28804717 x 10^{-13}
$nS_{1/2}$ <-> $2P_{1/2}$	-8.15820025 x 10^{-13}	7.87117712 x 10^{-13}
$nD_{3/2}$ <-> $2P_{1/2}$	-8.90760434 x 10^{-13}	9.79578024 x 10^{-13}
$nD_{5/2}$ <-> $2P_{1/2}$	-7.82118938 x 10^{-13}	7.13946213 x 10^{-13}
$nS_{1/2}$ <-> $2P_{3/2}$	-2.35573711 x 10^{-12}	1.3059391 x 10^{-12}
$nD_{3/2}$ <-> $2P_{3/2}$	-2.43067839 x 10^{-12}	1.54557663 x 10^{-12}
$nD_{5/2}$ <-> $2P_{3/2}$	-2.32203562 x 10^{-12}	1.19560858 x 10^{-12}

Comparison with the Euclidean Dirac + QED results show these non-Euclidean models fair noticeably better. The sample average prediction errors and standard deviations are generally smaller, indicating an increase in accuracy and precision. These results would seem to support the assumption of the physical presence of stronger relativistic effects in hydrogen than just Euclidean special. The first jump to spherical Schwarzschild theory shows the general magnitude of these non-Euclidean effects. The electronic Schwarzschild time dilation is (eq. (1) in this paper)

$$\frac{dt}{d\tau} = \left(1 - \left(\frac{v_n}{c}\right)^2 - \frac{r_S}{r_n}\right)^{-1/2}$$

For hydrogen, $r_S \approx 10^{-15}$ m. The ground state radius of hydrogen equals about 10^{-11} m. The ratio $r_S / r_n \approx 10^{-4}$, essentially as significant as introducing Euclidean special relativity itself , since $(v_n / c)^2 \approx 10^{-4}$.

In these non-Euclidean equations, exclusive use of their simplifications to “circular, equatorial” forms has been utilized. This is allowed, even for “P” or “D” of “F”, etc., “elliptical” subshells. The reason why, is how “ellipticity” is represented in Kerr theory. The Kerr field itself “goes elliptical” when the frame dragging *a* is not zero. Completely conserved circular and equatorial relativistic Kerr geodesics are possible, even in the aspherical field. In classic Sommerfeld theory, with its assumed spherical symmetry of the potential (as in Dirac theory), “ellipticity” demanded elliptical orbits. But in this generalized orbit theory, it’s the field that “goes elliptical” while the orbit itself remains perfectly circular, with conservation of the (shifted) relativistic Kerr total circular orbital energy and angular momentum. Such is the amazing “plasticity” of a Kerr geometry, and the generalization of the field geometry used for atomic modeling to Kerr seems to produce predictions closer to nature.

Hydrogen's hyperfine splitting can be inserted into the Kerr theory in an easy manner. The (electron spin)-(proton spin) hyperfine orientation delta-energy contributing to the electron's total orbital energy is

$$\Delta E_{hf} = \pi\hbar\left\langle \frac{\hat{A}\hbar}{2\pi} \right\rangle [F(F+1) - I(I+1) - j(j+1)] \tag{14}$$

where the quantum numbers F and I are the (Fermi) proton spin related quantum numbers. The value of I is always ½, and for n = 1,2, F can take on the two values of 0 and 1. For hydrogen's ground state (n = 1), the value of $< \hat{A}\hbar / 2\pi >$ can be obtained very accurately from experimentation, and equals $< \hat{A}\hbar / 2\pi > \; = 1420.405751768\,\text{Mhz}$

The spin-spin hyperfine delta energy can be easily incorporated in the Kerr theory by simply summing it with the spin-orbit and QED delta energies. The (negative of the) total (sum of the) spin-orbit, QED and spin-spin (hyperfine) delta energy is set equal to the constant β_0 term in solving for the electromagnetic Kerr frame dragging parameter a:

$$\beta_0 = -(\Delta E_{so} + \Delta E_{QED} + \Delta E_{hf})$$

Upon root solving, this produces an electromagnetic Kerr frame dragging parameter $a_{n,j,l,s,F,I}$ which is now a function of all of hydrogen's quantum numbers, Dirac and Fermi, and with Euclidean QED effects incorporated.. The hyperfine splittings were programmed into the electromagnetic Kerr transition wavelength predictions, and due to their low value of energy shifts, they did not produce significantly different results as compared to those last reported.

Conclusion, and Further Theory

The following conclusion seems inescapable: There exists stronger non-Euclidean relativistic effects in hydrogen as compared to what is predicted by Euclidean Dirac theory. The better agreement of the predictions by the non-Euclidean Kerr field theory with the observed data suggest the electron is experiencing a greater amount of time dilation as previously thought. The increase in time dilation, above and beyond Euclidean theory, is due to not only the electron's velocity (v), but also its position (r) in a generalized atomic electromagnetic Kerr field.

Perhaps the entirety of Euclidean-based QM should be generalized to a more non-Euclidean field theory. The generalization starts with a generalization of Schroedinger's equation:

$$-\frac{\hbar^2}{2\mu}\nabla^2\psi(r,\theta,\phi)+V(r,v)\psi(r,\theta,\phi)=E(r,v)\psi(r,\theta,\phi) \tag{15}$$

The inserted energy functionality on both *r* and *v* sets up the jump to a more "generalized" geometry. To immediately generalize this differential equation for a bound two-body system, the total and potential energy equations are identified with the electromagnetic Kerr circular, equatorial equations:

$$E(r,v)=\frac{\mu c^2}{2}\left[\left(\left(1-\frac{r_S}{r}+r_S a\frac{v}{r^2}\right)\frac{dt}{d\tau}\right)^2-1\right] \tag{16}$$

and

$$V(r,v)=-\frac{r_S}{2r}+\frac{1}{2}v^2+\frac{1}{2}\left(1-E^2(r,v)\right)\left(1+\frac{a^2}{r^2}\right)-\frac{r_S}{2r^3}\left(vr-aE(r,v)\right)^2 \tag{17}$$

These energy equations are obviously more complicated than Euclidean theory (comes with being more "general"), but the tremendous simplicity of their circular, equatorial cases should reduce solution complexity. The results would naturally include magnetism as part of the metric structure, per the inclusion of *a* in the equations. Recall in non-Euclidean field theory, magnetism is model by a field warp, not an orbit shape shift, and circular equatorial orbit theory suffices, even when Kerr magnetic fields exist. Kerr-Schroedinger circular wave equation solutions could be found, and then Born probability introduced. If "maximum pdf/maxim mode" moments are used, essentially "deterministic orbit theory" is the result, and the equations presented here fall out from this generalized Kerr-Schroedinger-Born theory. As proven here, this generalization of forces in hydrogen produces predictions in better agreement with its observed Balmer spectral signature.

Appendix

The extended GEM Equivalence Principle (EP) allows setting the electronic force on the electron while in a subshell, equal to any other (equal) force, inertial or field induced. Bohr used this "extension" of Einstein's EP by equating the electronic force on the electron, to the electron's "inertial" force:

$$\frac{1}{4\pi\varepsilon_0}\frac{|e_e e_p|}{r^2} = m_e a \tag{A.1}$$

The right of (A.1) comes from simply "*F = ma*," but the force on the right is actually a field induced force. For extension of the simple Euclidean theory into the general non-Euclidean domain, the GEM EP allows deriving a type of "equivalent gravitational force model" for insertion on the right. The obvious candidate is

$$\frac{1}{4\pi\varepsilon_0}\frac{|e_e e_p|}{r^2} = G\frac{\chi e_p m_e}{r^2} \tag{A.2}$$

The right side of (A.2) is an "effective gravitational force." This effective force exactly equals the electronic force on the left. The "unification curvature parameter" χ is a mass-to-charge ratio which converts the proton's charge e_p into an amount of "effective mass" which produces exactly the same magnitude of electronic binding forces.

Solving for χ produces

$$\chi = \frac{1}{G4\pi\varepsilon_0}\frac{|e_e|}{m_e} \tag{A.3}$$

In the effective gravitational model, the effective central mass χe_p binds the electron's mass m_e into orbits allowed by the exact same magnitude of the actual electronic binding forces.

The simple extension into non-Euclidean geometries is made by utilizing the definition of the "gravitational radius," that is, the Schwarzschild radius in gravitational Schwarzschild theory. In gravitational theory this radius is $r_S = 2GM/c^2$ where *M* is the rest mass of the central body. Inserting the effective central mass $M = \chi e_p$ into this equation produces the electronic Schwarzschild radius, devoid of *G*:

$$r_S = \frac{1}{2\pi\varepsilon_0}\frac{\left|e_e e_p\right|}{m_e c^2} \tag{A.4}$$

This “characteristic length” then enters the elements of a Schwarzschild metric tensor, defining a tightly-curved, atomic-sized electronic Schwarzschild field.